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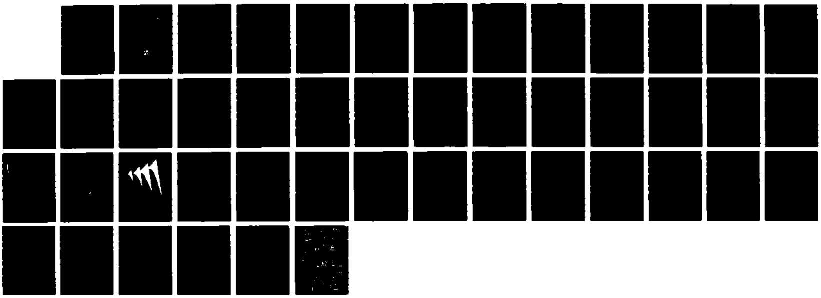

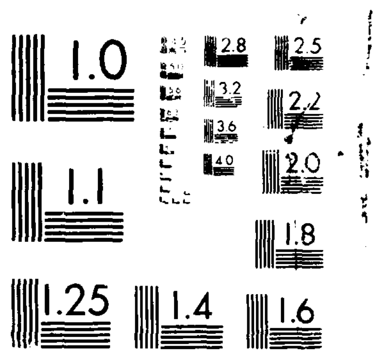

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DIRICHLET PRIOR IN BAYESIAN ESTIMATION OF ITEM RESPONSE CURVES

ROBERT K. TSUTAKAWA

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This article examines the use of the ordered Dirichlet prior for binary logistic item response models. This prior is based on the investigator's prior information about the probabilities of correct response to items from examinees at several ability levels. The effect of the prior is examined in terms of the posterior mode of item parameters computed via the EM algorithm. An illustration describes the application of a 1981 ACT math test to form a prior which is used on a similar 1987 test. Detailed computational expressions for the three-parameter logistic model are summarized in an appendix.

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DIRICHLET PRIOR IN BAYESIAN ESTIMATION OF ITEM RESPONSE CURVES

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Introduction

One method of formally incorporating prior opinion and partial information into the estimation of latent trait models is the Bayesian approach. The implementation of this approach to practical problems is made difficult due to the lack of tools to quantitatively deal with prior information. The purpose of this paper is to examine some tools to facilitate the selection of prior distribution and to demonstrate how this distribution can be used to estimate models for mental testing.

It will be assumed that the responses to test items are dichotomous (correct or incorrect) and that each item can be characterized by an item response curve, a function of ability indexed by unknown parameters, called item parameters. Most of the discussions will be on the three–parameter logistic (3PL) curve (Birnbaum, 1968), with the focus on formulating a prior distribution and on computing the posterior mode of item parameters. The assumptions and techniques closely follow those in Tsutakawa & Lin (1986), where an illustration was given for the two–parameter logistic (2PL) with a prior which differs from the one in this paper.

A standard method of estimating ability and item parameters for 3PL is maximum likelihood (ML). This approach has been extensively discussed by Lord (1980) and become quite popular since the availability of a number of convenient computer programs such as LOGIST (Wingersky, Barton & Lord, 1982). Under the assumption that the abilities are randomly sampled from some population distribution, the marginal maximum likelihood (MML) estimation of item parameters has been discussed by Bock and Aitkin (1981) and Ridgon & Tsutakawa (1983), among others.

Following the argument developed for hierarchical linear models by Lindley & Smith (1971), Swaminathan & Gifford (1986) have proposed the use of the joint posterior mode of ability and item parameters for 3PL. Mislevy & Bock (1985) and Tsutakawa & Lin (1986) use the EM algorithm (Dempster, Laird & Rubin, 1977) to compute the posterior mode of item parameters for 3PL and 2PL, respectively. The priors used by

Swaminathan & Gifford and Mislevy & Bock assume independence among item parameters, not only between but within items. In Tsutakawa & Lin, the dependence among parameters within items is introduced into the prior through the use of an ordered bivariate beta distribution for values of the item response curve at two ability levels. Mislevy (1986) proposed representing such dependence via multivariate normal priors on the item parameters. The proper representation of the joint prior distribution of parameters within items is particularly important in the presence of preliminary information or previous data on the items.

In this paper, the Tsutakawa & Lin prior is modified by replacing the ordered beta by an ordered Dirichlet. The advantage of this prior is that it has few parameters and simpler to select since the marginal distributions are betas. The Dirichlet distribution has been extensively studied in Wilks (1962) and used as prior distribution by Ramsey (1972) for quantal response functions in bioassay and Ferguson (1973) for nonparametric inference. The use of this distribution here is more limited. It is used to facilitate incorporating prior information about items which may be more readily available in terms of response probabilities rather than item parameters.

The paper begins with a statement of the general problem and a discussion of some difficulties encountered in implementing Bayesian principles. To facilitate the selection of prior distributions a reparameterization of the item parameters is introduced. This is followed by an examination of the Dirichlet distribution as a means of specifying the prior. The method is adapted to estimating 3PL curves for a 1987 American College Testing Program (ACT) math test, with prior distribution based on a distribution of 3PL curves from a 1981 ACT math test. The robustness of the choice of prior is illustrated in terms of changes in the posterior modes as the amount of weight placed on the prior is varied. These estimates are numerically compared to MML and LOGIST estimates. A plot of the estimated 3PL curves shows how the Bayes esitmates are shrunk towards the prior mean relative to either MML and LOGIST. This has the effect of preventing the occurence of extreme outcomes frequently encountered by maximum likelihood methods. The extensive computational expressions required to implement the EM algorithm are summarized in the Appendix.

General Setup and Problems

Consider each of n examinees responding to a test with k items. Let $y_{ij} = 0$ or 1 according as the response to item j by examinee i is incorrect or correct. Assume the probability of a correct response to an item is given by an item response function $p_\xi(\theta)$ depending on the unknown item parameter ξ and real valued ability θ. For 3PL, which will be discussed below, this function has the form

$$p_\xi(\theta) = c + \frac{1-c}{1+\exp\{-a(\theta-b)\}}\ , \tag{1}$$

for $-\infty < \theta < \infty$, where $\xi = (a, b, c)$, $0 < a$, $-\infty < b < \infty$, and $0 < c < 1$. The parameter space for ξ will be denoted by Ω.

Given k items with parameters $\boldsymbol{\xi} = (\xi_1,...,\xi_k)$ and n individual with abilities $\boldsymbol{\theta} = (\theta_1,...,\theta_n)$, assume conditional independence among the responses so that the joint probability of the n×k matrix $\mathbf{y} = ((y_{ij}))$ is given by

$$P(\mathbf{y}|\boldsymbol{\xi},\boldsymbol{\theta}) = \prod_i \prod_j P(y_{ij}|\xi_j,\theta_i), \tag{2}$$

where

$$P(y_{ij}|\xi_j,\theta_i) = p_{\xi_j}(\theta_i)^{y_{ij}}\{1 - p_{\xi_j}(\theta_i)\}^{1-y_{ij}},\ y_{ij} = 0,1.$$

Moreover assume that $\theta_1,...,\theta_n$ are iid N(0,1). [Without loss of generality, N(0,1) is used rather than $N(\mu,\sigma^2)$ with (μ,σ^2) unknown in order to avoid the indeterminacy in the parameterization associated with the 3PL model (Lord, 1980, pp. 36–38).]

The problem is to estimate ξ based on the data **y** when there is previous information about the items. If this information can be summarized in terms of a prior pdf $p(\xi)$ of ξ, Bayesian principles suggest that one should consider the marginal posterior distribution of ξ, given by

$$p(\xi|y) \propto p(\xi)\prod_i \int \prod_j P(y_{ij}|\xi_j,\theta_i)\varphi(\theta_i)d\theta_i \tag{3}$$

where $\varphi(\theta_i)$ is the N(0,1) pdf.

Having accepted this general principle, the major obstacles to carrying out the Bayesian approach are two–fold. The first obstacle is the selection of the prior or $p(\xi)$. The second is carrying out the computation in order to summarized the information about ξ after observing **y**. With the influx of high speed computing and the availability of numerical approximations, the second problem is becoming less crucial, though far from solved. The solution to the first problem is largely subjective and not adequately discussed. This paper is primarily on techniques for dealing with the first problem.

Reparameterization for 3PL

At three fixed ability levels $t_1 < t_2 < t_3$ consider the values of the 3PL curve

$$\mathbf{p}(\xi) = (p_1,p_2,p_3), \tag{4}$$

where

$p_i = c + \frac{1-c}{1+\exp\{-a(t_i-b)\}},$ $i = 1,2,3$. Now consider the space $\mathcal{P}$ spanned by $\mathbf{p}(\xi)$, i.e., $\mathcal{P} = \{\mathbf{p}|\mathbf{p} = \mathbf{p}(\xi) \text{ for some } \xi\epsilon\Omega\}$.

The Jacobian of this transformation is defined by the determinant

$$J = \begin{vmatrix} \partial p_1/\partial a & \partial p_1/\partial b & \partial p_1/\partial c \\ \partial p_2/\partial a & \partial p_2/\partial b & \partial p_2/\partial c \\ \partial p_3/\partial a & \partial p_3/\partial b & \partial p_3/\partial c \end{vmatrix}, \tag{5}$$

which may be simplified to

$$J = a(1-c)^2 \exp(ab)(t_3-t_1) \prod_{i=1}^{3} \varphi_{t_i} \psi_{t_i} \{w_{12}\exp(-at_3) + w_{23}\exp(-at_1) - \exp(-at_2)\}, \tag{6}$$

where $w_{12} = (t_2-t_1)/(t_3-t_1)$, $w_{23} = (t_3-t_2)/t_3-t_1)$, $\varphi_{t_i} = [1 + \exp\{-a(t_i-b)\}]^{-1}$ and $\psi_{t_i} = 1 - \varphi_{t_i}$. The fact that $J > 0$ for all $(a,b,c) \epsilon\ \Omega$, may be seen by noting that all factors outside the brackets {} are positive and that the expression in {} is positive since the sum of the first two terms is a weighted average of values of a convex function which is larger than the third term, the value of this function at t_2 (the weighted average of t_1 and t_3). It follows that the transformation (4) is nonsingular and the 3PL curves may be parameterized in terms of $\mathbf{p} \epsilon \mathscr{P}$.

Now let $\mathscr{O}$ denote the set of all triples $\mathbf{p} = (p_1,p_2,p_3)$ with $0 < p_1 < p_2 < p_3 < 1$. Inspite of the richness of the 3PL family, not all points in $\mathscr{O}$ belong to $\mathscr{P}$. The following result is useful in characterizing the points in $\mathscr{P}$.

Theorem. Given $t_1 < t_2 < t_3$ and any $\mathbf{p} \epsilon \mathscr{O}$, there exists a point $\xi \epsilon \Omega$ such that $\mathbf{p} = \mathbf{p}(\xi)$ if and only if there exist some c such that $0 < c < p_1$ and

$$(L_2-L_1)/(t_2-t_1) = (L_3-L_2)/t_3-t_2), \tag{7}$$

where

$$L_i = \ln\{(p_i-c)/(1-p_i)\}, \quad i=1,2,3, \tag{8}$$

Proof: Given $(a,b,c) \epsilon \Omega$, the equation (4) may be rewritten

$$L_i = a(t_i-b), \quad i = 1,2,3. \tag{9}$$

But (9) implies (7) and (4) implies $0 < c < p_1$.

Conversely, given $(p_1, p_2, p_3) \epsilon \mathcal{O}$ suppose there exists a c such that $0 < c < p_1$ and (7) holds. For this c, a and b may be solved from (9) and are given by

$$\begin{aligned} a &= (L_2-L_1)/(t_2-t_1), \\ b &= t_1 - L_1/a \ . \end{aligned} \tag{10}$$

Note that $a > 0$ since $c < p_1 < p_2$ and $L_2 > L_1$. Thus the resulting $\xi = (a,b,c) \epsilon \Omega$ and $p(\xi) = \mathbf{p}$.

An important special case in which one can explicitly find ξ for a given $\mathbf{p} \epsilon \mathcal{P}$ is where the t_i are equally spaced. The solution is given by the following.

Corollary. Given $\mathbf{p} \epsilon \mathcal{P}$ and $t_3-t_2 = t_2-t_1 > 0$, c is given by

$$c = \frac{-\beta \pm (\beta^2-4\alpha\gamma)^{1/2}}{2\alpha}, \tag{11}$$

where

$$\alpha = (1-p_1)(1-p_3) - (1-p_2)^2,$$
$$\beta = 2p_2(1-p_1)(1-p_3) + (p_1+p_3)(1-p_2)^2,$$
$$\gamma = p_2^2(1-p_1)(1-p_3) - p_1p_3(1-p_2)^2.$$

Proof. If $t_3-t_2 = t_2-t_1 > 0$, (7) simplifies to

$$(p_2-c)^2/(1-p_2)^2 = \{(p_1-c)/(1-p_1)\}\{(p_3-c)/(1-p_3)\} \tag{12}$$

This is a quadratic equation in c whose solution is given by the Corollary. It is immediately seen by inspection that one of the roots is always equal to c=1 and may be ignored since we must have $c < p_1 < 1$. Once c is available a and b may be obtained from (10).

As an example of a point **p** in $\mathcal{O}$ not in $\mathcal{P}$, consider $(t_1, t_2, t_3) = (-1, 0, 1)$ and $(p_1, p_2, p_3) = (.05, .50, .55)$. In this case $(\alpha, \beta, \gamma) = (.1775, -.2775, .0100)$ and c = .56 and 1. Since $c > p_1$, $\mathbf{p} \notin \mathcal{P}$.

Constrained Dirichlet Prior

Following Tsutakawa & Lin (1986), consider prior information about the item response curves at fixed ability levels rather than about item parameters directly. Then consider the distribution induced on the item parameters through the transformation relating values of the item response function to the item parameters.

For a given item let $p_1,\ldots,p_m$ be the probabilities of correct responses at predetermined ability levels $t_1 < t_2 <\ldots< t_m$. Now define the increments

$$\begin{aligned} x_1 &= p_1, \\ x_2 &= p_2 - p_1, \\ &\vdots \\ x_{m+1} &= 1 - p_m . \end{aligned} \tag{13}$$

Suppose our prior information about these increments can be represented by the m – dimensional Dirichlet distribution with pdf,

$$f(x_1,\dots,x_m) = \frac{\Gamma(N)}{\prod_{s=1}^{m+1} \Gamma(\pi_s N)} x_1^{\pi_1 N-1} x_2^{\pi_2 N-1} \dots (1-x_1-\dots-x_m)^{\pi_{m+1} N-1} \tag{14}$$

$0 < x_i <\dots< x_m < 1$, where $(\pi_1,\dots,\pi_m, N)$ are parameters such that $\pi_s > 0$, $\sum_{s=1}^{m+1} \pi_s = 1$, and $N > 0$. The first two moments of (14) are given by

$$\begin{aligned} E(x_s) &= \pi_s, \\ Var(x_s) &= \pi_s(1-\pi_s) / (N+1), \\ Cov(x_r, x_s) &= -\pi_r \pi_s/(N+1), \quad \text{for } r \neq s. \end{aligned} \tag{15}$$

Under the transformation (4), $(p_1,\dots,p_m)$ has the ordered Dirichlet distribution *(Wilks 1962) with pdf*

$$g(p_1,\dots,p_m) = \frac{\Gamma(N)}{\prod_{s=1}^{m+1} \Gamma(\pi_s N)} p_1^{\pi_1 N-1} (p_2-p_1)^{\pi_2 N-1} \dots (1-p_m)^{\pi_{m+1} N-1}, \tag{16}$$

for $0 < p_1 <\dots< p_m<1$. Because of the well known "lumping" property of the Dirichlet, $p_s = x_1+\dots+x_s$ will have a marginal distribution which is beta. From (15), the first two moments of $(p_1,\dots,p_m)$ are

$$\begin{aligned} \mu_s &= E(p_s) = \pi_1+\dots+\pi_s, \\ \sigma_s^2 &= Var(p_s) = \mu_s(1-\mu_s)/(N+1), \end{aligned} \tag{17}$$

for $1 \leq s \leq m$, and

$$\sigma_{rs} = \mathrm{Cov}(p_r,p_s) = \mu_s(1-\mu_r)/(N+1) - \mu_r(\mu_s-\mu_r)/(N+1),$$

for $1 \leq r < s \leq m$.

Now the specification of $(\mu_1,\ldots,\mu_m,N)$, $\mu_s < \mu_{s+1}$, will uniquely define the value for $(\pi_1,\ldots,\pi_m,N)$ and hence the joint distribution for $(p_1,\ldots,p_m)$. In practice the μ's can be chosen to represent the prior point estimate of the p's and N to represent the weight of the prior or tightness of the prior about the estimate as expressed in the variances σ_s^2. Further discussion on selection will be given in the example.

For the application to 3PL, consider the case m=3 and the constrained distribution with pdf

$$g_C(p_1,p_2,p_3) = K \frac{\Gamma(N)}{\prod_{s=1}^{4} \Gamma(\pi_s N)} \prod_{s=1}^{4} p_s^{\pi_s N-1}, \tag{18}$$

for $(p_1,p_2,p_3) \epsilon \mathscr{P}$ and 0 otherwise, where K is a normalizing constant. (See Box & Tiao, 1973, p. 67 for a general discussion on constrained distributions.) This distribution induces a distribution for $\xi \epsilon \Omega$ through the transformation (4). The induced distribution will have pdf

$$p(a,b,c) = |J| g_C(p_1(\xi),\, p_2(\xi),\, p_3(\xi))$$

for $\xi \epsilon \Omega$, where J is defined by (6) and $(p_1(\xi), p_2(\xi), p_3(\xi)) = \mathbf{p}(\xi)$.

There is some problem in selecting $(\pi_1, \pi_2, \pi_3, \pi_4, N)$ since the moments under g_C will differ from those under g. This will not be an important issue when N is large since the continuity of the transformation assures us that a tight distribution for **p** will induce a tight distribution for ξ and the discrepancy between g and g_C will be negligible. The numerical work in the next section suggests that one may approximate g_C by g for moderate size N.

Formulating a Prior from Previous Data

The application of previous information to analyze new data will be illustrated here and in the next section in term of two ACT math tests. The data from the 1981 test will be use to form a prior distribution for item parameters used to estimate 3PL curves for the 1987 test.

Figure 1 gives 40 3PL curves estimated by LOGIST based on a random sample of n=2000 from the 1981 test. Considering the year to year similarity in ACT tests and the limited information about the individual items in the 1987 test, it seems reasonable to assume that the sample characteristics of 3PL curves for 1987 will be similar to those for 1981. One might reason, in this case, that the 1987 curves will behave like a random sample from the same distribution of curves that produced those for 1981. If additional information is available for specific items the random sample assumption would be unreasonable and different priors should be formed for different items. There is nothing that precludes the use of subjective opinion at this stage.

In order to select the parameters for the Dirichlet distribution, the three levels chosen for this example are $(t_1, t_2, t_3) = (-1{,}28, 0, 1.28)$ corresponding to the (10, 50, 90) percent points of the normal distribution. The sample averages $\bar{p}_j$ and standard deviations s_j of the values of the curves in Figure 1 at these three point are tabulated in Table 1.

Consider matching these moments to the moments (17) of the ordered Dirichlet. Since the variances cannot be matched at the three levels by a single N, the average of the three N's obtained from three separate fittings is used. The average so computed is $\bar{N} = (16.2 + 6.6 + 5.6)/3 = 9.8$. The 3PL curve passing through the points $(t_1,\bar{p}_1)$, $(t_2,\bar{p}_2)$,$(t_3\bar{p}_3)$ and the resulting marginal beta distributions of p_1, p_2, p_3 are plotted in

Figure 2.

The relative position of $\mathscr{P}$ to $\mathscr{O}$ for this example is sketched in Figure 3. In order to relate the unconstrained to the constrained Dirichlet, points in $\mathscr{O}$ were randomly simulated and tested for inclusion in $\mathscr{P}$ by the criterion stated in the Theorem. Of the 1,000 points thus simulated 81% were also in $\mathscr{P}$ and had the sample means and standard deviations tabulated in Table 1. The discrepancy between the distribution in $\mathscr{P}$ and $\mathscr{O}$ averages about 4% in terms of the means and 5% in terms of the standard deviations.

In order to examine the effect of N on the moments, the simulation was repeated for N=3 and 24. Of the 1,000 points 70% were in $\mathscr{P}$ for N=3 and 88% for N=24. The sample results in Table 1 indicate the increased similarity between the Dirichlet and constrained Dirichlet as N increases. The results also suggest the possibility for finding a value of N for which the sample moments obtained from the ACT curves will be reasonably matched to those of the constrained Dirichlet. It will be shown in the next section that the estimated 3PL curves are quite robust with respect to the choice of N and that a precise specification of N is not crucial.

Posterior Mode for 1987 ACT

The prior based on the 1981 test will now be used to estimate k=40 3PL curves for the 1987 test with a random sample of n=400.

The item parameters will be changed to ξ = (b,c,d) where d = log a or a = exp(d) in (1). This reparameterization is made to enhance the asymptotic normality of the posterior distribution and to speed up the convergence of the EM algorithm. (Similar strategies have been suggested by Naylor & Smith, 1982 and by Mislevy, 1986.)

The posterior mode and marginal maximum likelihood (to be denoted by MLF3 for 3PL) estimates of the item parameter ξ were computed via the EM algorithm. Computational expressions are summarized in the Appendix. Figures 4, 5, and 6 give scatter plots of the b,c, and d parameters. The two estimates of the b parameter are generally quite close except in the lower range where the Bayes estimates shows more shrinkage toward the average. This type of shrinkage is more pronounced for the c parameters, where 5 items had c parameters which were positive under Bayes but zero under MLF3. The estimates of the d parameters were also fairly close except for the 3 items with large d values for MLF3.

To study the effect of the prior on the posterior mode two additional cases corresponding to N=3 and 24, discussed in the last section, were considered. Figures 7 through 10 show a sample of estimated 3PL curves under MLF3, LOGIST and Bayes for N=3, 9.8, 24. They also show the prior mean to illustrate the amount of attraction towards the prior means as a function of N.

Item 27, shown in Figure 7, exhibits a pattern where all estimates are mutually close and close to the prior mean. Item 7, shown in Figure 8, exhibits a more typical pattern, shared by most items, where the estimated curves are fairly close, but with the Bayes estimates, particularly those with N large, being closer to the prior mean than either MLF3 or LOGIST. Item 13, shown in Figure 9, is the case showing the largest discrepancy, particularly with respect to slope. The instability of the LOGIST estimate was indicated by the low value of $b-2(a/1.7)$, which was given as -13.01 for this item. The instability appears to be due to the difficulty of the item which, in turn, caused a considerable amount of guessing. Item 22, shown in Figure 10, exhibits a moderately close agreement for the central and upper θ values, although there are notable differences among the slopes and lower asymptotes.

Although it is difficult to make general statements based on the limited data studied here, the results confirm certain properties found in related studies. There is general agreement among the estimates for most items. When disagreement exists it tend to occur where non–Bayesian estimates take on extreme values. The choice of N is not too essential and appears more crucial in situations where the non–Bayesian estimates are least stable.

Discussion

The primary purpose of this paper has been to demonstrate the Bayesian use of previous information to analyze new item response data. Discussions on the advantages and disadvantages of Bayesian methods may be found in Lord (1986), Mislevy (1986), Swaminathan & Gifford (1986) and Tsutakawa & Lin (1986) and will not be repeated here.

The purpose of working through the Dirichlet is to facilitate the formulation of a prior distribution in terms of measurements more familiar to the user. Although the illustration here has been limited to an exchangeable prior, the method may be easily modified to situations where prior information varies from item to item. This would include instances where items are assembled from several sources, including those where items have been previously analyzed under different models, e.g., 2PL.

One technical problem associated with the current approach is the difference between the space $\mathscr{P}$ spanned by the 3PL curves and the space $\mathscr{O}$ of the ordered Dirichlet. Although this is not a problem when N is large , some rule of thumb adjustment would be desirable for small N. The robustness of the posterior mode suggests that a precise value will not be unnecessary.

One limitation of the Dirichlet prior is that it has only one parameter, N, to express the tightness of the prior distribution. In cases where there is considerable variability in information from one t_i to another, the use of the ordered beta (Tsutakawa & Lin, 1986) would seem preferable. This situation could arise, for example, when items have been previously used on a group of individuals whose ability levels are generally lower (or higher) than those of the current examinees.

The use of the prior distribution here was limited to obtaining the posterior mode, a point estimate of the item parameter. A more important use would be to analyze the posterior uncertainty in both the item and ability parameters. Such uncertainties in the item parameters can be evaluated by the posterior covariance matrix, which can be approximated by the inverse of the posterior information matrix (Tsutakawa & Lin, 1986). The posterior variances of the ability parameters are more difficult to work with but may be approximated by extending the approach by Tsutakawa & Soltys (1988) for 2PL.

Appendix: Computational Expressions for the EM Algorithm

As shown in Tsutakawa and Lin (1986), the key steps of the EM algorithm in finding the posterior mode of ξ may be summarized as follows.

Starting with some initial approximation ξ° to the mode, maximize separately for each j the functions

$$\sum_{i=1}^{n} \int \log P(y_{ij}|\theta_i,\xi_j) p(\theta_i|y_i,\xi^{\circ}) d\theta_i + \log p(\xi_j), \tag{A.1}$$

where $\mathbf{y}_i = (y_{i1},...,y_{ik})$, $p(\theta_i|y_i,\xi^{\circ})$ is the posterior pdf of θ_i when ξ is known and equals ξ°, and $p(\xi_j)$ is the prior for the jth item parameter, $j = 1,...,k$. Then iterate the maximization of these function after replacing ξ° by the value of $\xi = (\xi_1,...,\xi_k)$ which maximized the function at the last iteration. The iteration is repeated till some convergence criterion is satisfied.

The maximizations require numerical integration and some iterative procedure such as the one by Marquardt (1963), which is used here. Marquardt's procedures requires the evaluation of the first and second partial derivatives of (A.1) with respect to $\xi_j = (b_j,c_j,d_j)$.

Since the maximization is carried out separately for each j, the subscript j will be dropped and notations

$$Z = \sum_i \int \log P(y_{ij}|\theta_i,\xi_j) p(\theta_i|y_i,\xi^{\circ}) d\theta_i$$

and

$$g\,(i,\theta) = \log P(y_{ij}|\theta_i,\xi_j)$$

will be used. Now denote the first and second partials of Z and $g(i,\theta)$ with respect to $(b,c,d) = (b_j,c_j,d_j)$ by Z_u, Z_{uv}, $g_u(i,\theta)$ and $g_{uv}(i,\theta)$ for $u,v = 1,2,3$ so that, for example, $Z_2 = \partial Z/\partial c$ and $g_{13}(i,\theta) = \partial^2 g(i,\theta)/\partial b \partial d$.

To simplify the notation define

$\varphi_\theta = \{1 + \exp[-\exp(d)(\theta - b)]\}^{-1}$,

$\psi_\theta = 1 - \varphi_\theta$,

$\lambda_\theta = \{1 + c\exp[-\exp(d)(\theta - b)]\}^{-1}$,

$\eta_\theta = \{c + \exp[\exp(d)((\theta - b)]\}^{-1}$.

Then the derivatives of $g(i,\theta)$ may be expressed as follows.

$$g_1(i,\theta) = -\exp(d)(y_{ij}\lambda_\theta - \varphi_\theta),$$

$$g_2(i,\theta) = y_{ij}[\eta_\theta - 1/(c-1)] + 1/(c-1),$$

$$g_3(i,\theta) = (\theta - b)\exp(d)(y_{ij}\lambda_\theta - \varphi_\theta),$$

$$g_{11}(i,\theta) = \exp(2d)\{cy_{ij}\lambda_\theta\eta_\theta - \varphi_\theta\psi_\theta\},$$

$$g_{12}(i,\theta) = y_{ij}\exp(d)\eta_\theta\lambda_\theta,$$

$$g_{13}(i,\theta) = -\exp(d)(y_{ij}\lambda_\theta - \varphi_\theta) + \exp(2d)(\theta - b)[\varphi_\theta\psi_\theta - cy_{ij}\lambda_\theta\eta_\theta],$$

$$g_{22}(i,\theta) = -[y_{ij}\eta_\theta^2 + (1 - y_{ij})/(c-1)^2],$$

$$g_{23}(i,\theta) = -\exp(d)(\theta - b)y_{ij}\lambda_\theta\eta_\theta,$$

$$g_{33}(i,\theta) = (\theta - b)^2\exp(2d)\{cy_{ij}\lambda_\theta\eta_\theta - \varphi_\theta\psi_\theta\} + (\theta - b)\exp(d)\{y_{ij}\lambda_\theta - \varphi_\theta\}.$$

The derivatives of Z are given by

$$Z_u = \sum_{i=1}^{n} \bar{g}_u(i),$$

and

$$Z_{uv} = \sum_{i=1}^{n} \bar{g}_{uv}(i),$$

where

$$\bar{g}_u(i) = \int_{-\infty}^{\infty} g_u(i,\theta)p(\theta|y_i,\xi^{\circ})d\theta$$

and

$$\bar{g}_{uv}(i) = \int_{-\infty}^{\infty} g_{uv}(i,\theta)p(\theta|y_i,\xi^{\circ})d\theta.$$

See Tsutakawa (1984) for scaling techniques and Gauss–Hermite approximations of these intergrals.

Again suppressing the subscript j, the prior for the jth item parameter is given by

$$p(b,c,d) \propto |J| p_1^{\nu_1-1}(p_2-p_1)^{\nu_2-1}(p_3-p_2)^{\nu_3-1}(1-p_3)^{\nu_4-1}$$

where $\nu_i = \pi_i N$,

$$p_i = c + \frac{1-c}{1+\exp\{-\exp(d)(t_i-b)\}} ,$$

for i = 1,2,3 and J is the Jacobian given by

$$J = \exp(2d+be^d)(1-c)^2 \prod_{i=1}^{3} (\varphi_{t_i}\psi_{t_i})$$
$$\{(t_3-t_2)\exp(-t_1e^d)+(t_2-t_1)\exp(-t_3e^d)-(t_3-t_1)\exp(-t_2e^d)\}.$$

J can be shown to be positive and its expression differs form (6) since the parameterization is different. When $t_2 = 0$, which is the case used in the numerical examples, there is a slight simplification in the expressions for the log prior and its first two partial derivatives which may be given as follows.

$$\log p(b,c,d) = \text{constant}$$
$$+ 2d + be^d + 2\log(1-c) + \sum_{i=1}^{3}(\log\varphi_{t_i}+\log\psi_{t_i}) + \log\{t_1(1-f_3) - t_3(1-f_1)\}$$
$$+ \sum_{i=1}^{4}(\nu_i-1)\log(p_i - p_{i-1}) ,$$

where $f_i = \exp(-e^d t_i)$, for i = 1,3, $p_0 = 0$, and $p_4 = 1$.

$$\partial\log p(b,c,d)/\partial b = e^d + \sum_{i=1}^{3} e^d(\varphi_{t_i}-\psi_{t_i}) + \sum_{i=1}^{4}(\nu_i-1)\partial\log(p_i-p_{i-1})/\partial b,$$

$$\partial \log p(b,c,d)/\partial c = 2/(c-1) + \sum_{i=1}^{4}(\nu_i-1)\partial \log(p_i-p_{i-1})/\partial c$$

$$\partial \log p(b,c,d)/\partial d = 2 + be^d + \frac{e^d t_1 t_3(f_3-f_1)}{t_1(1-f_3) - t_3(1-f_1)} + \sum_{i=1}^{3} e^d(t_i-b)(\psi_{t_i}-\varphi_{t_i})$$

$$+ \sum_{i=1}^{4}(\nu_i-1)\partial \log(p_i-p_{i-1})/\partial d$$

$$\partial^2 \log p(b,c,d)/\partial b^2 = -2e^{2d}\sum_{i=1}^{3}\varphi_{t_i}\psi_{t_i} + \sum_{i=1}^{4}(\nu_i-1)\partial^2 \log(p_i-p_{i-1})\partial b^2,$$

$$\partial^2 \log p(b,c,d)/\partial c^2 = -2/(1-c)^2 + \sum_{i=1}^{4}(\nu_i-1)\partial^2 \log(p_i-p_{i-1})/\partial c^2,$$

$$\partial^2 \log p(b,c,d)/\partial d^2 = be^d$$

$$+ \frac{e^d t_1 t_3\{f_3(1-t_3e^d)-f_1(1-t_1e^d)\}\{t_1(1-f_3)-t_3(1-f_1)\}-\{e^d t_1 t_3(f_1-f_3)\}^2}{\{t_1(1-f_3) - t_3(1-f_1)\}^2}$$

$$+ \sum_{i=1}^{3}\{e^d(t_i-b)(\psi_{t_i}-\varphi_{t_i}) - 2e^{2d}(t_i-b)^2\varphi_{t_i}\psi_{t_i}\} + \sum_{i=1}^{4}(\nu_i-1)\partial^2 \log(p_i-p_{i-1})/\partial d^2,$$

$$\partial^2 \log p(b,c,d)/\partial b\partial c = \sum_{i=1}^{4}(\nu_i-1)\partial^2 \log(p_i-p_{i-1})/\partial b\partial c,$$

$$\partial^2 \log p(b,c,d)/\partial b\partial d = e^d + \sum_{i=1}^{3}\{e^d(\varphi_{t_i}-\psi_{t_i}) + 2e^{2d}(t_i-b)\varphi_{t_i}\psi_{t_i}\}$$

$$+ \sum_{i=1}^{4}(\nu_i-1)\partial^2 \log(p_i-p_{i-1})/\partial b\partial d,$$

$$\partial^2 \log p(b,c,d)/\partial c\partial d = \sum_{i=1}^{4}(\nu_i-1)\partial^2 \log(p_i-p_{i-1})/\partial c\partial d.$$

In order to complete the computational expressions for the derivatives of the log prior the first two derivatives of $\log(p_i-p_{i-1})$, are needed. Using the notation $\beta_1 = b$, $\beta_2 = c$, $\beta_3 = d$, these derivatives are given by

$$\frac{\partial}{\partial\beta_r}\log(p_i-p_{i-1}) = \left[\frac{\partial p_i}{\partial\beta_r} - \frac{\partial p_{i-1}}{\partial\beta_r}\right]/(p_i-p_{i-1}),$$

$$\frac{\partial^2}{\partial\beta_r\partial\beta_s}\log(p_i-p_{i-1})$$

$$= \left\{\left[\frac{\partial^2 p_i}{\partial\beta_r\partial\beta_s} - \frac{\partial^2 p_{i-1}}{\partial\beta_r\partial\beta_s}\right](p_i-p_{i-1}) - \left[\frac{\partial p_i}{\partial\beta_s} - \frac{\partial p_{i-1}}{\partial\beta_s}\right]\left[\frac{\partial p_i}{\partial\beta_r} - \frac{\partial p_{i-1}}{\partial\beta_r}\right]\right\}/(p_i-p_{i-1})^2,$$

r,s = 1,2,3. For i=0 and 4 the derivatives of p_i are zero. For i = 1,2,3, they are

$$\partial p_i/\partial b = -e^d(1-c)\varphi_{t_i}\psi_{t_i},$$
$$\partial p_i/\partial c = 1 - \varphi_{t_i},$$
$$\partial p_i/\partial d = e^d(1-c)(t_i-b)\varphi_{t_i}\psi_{t_i},$$
$$\partial^2 p_i/\partial b^2 = e^{2d}(1-c)\{\varphi_{t_i}\psi^2_{t_i} - \varphi^2_{t_i}\psi_{t_i}\},$$
$$\partial^2 p_i/\partial c^2 = 0,$$
$$\partial^2 p_i/\partial d^2 = e^d(1-c)(t_i-b)\varphi_{t_i}\psi_{t_i}\{1 + e^d(t_i-b)(\psi_{t_i}-\varphi_{t_i})\}.$$
$$\partial^2 p_i/\partial b\partial c = e^d\varphi_{t_i}\psi_{t_i},$$
$$\partial^2 p_i/\partial b\partial d = -e^d(1-c)\phi_{t_i}\psi_{t_i}\{1 + e^d(t_i-b)\psi_{t_i} - e^d(t_i-b)\phi_{t_i}\},$$
$$\partial^2 p_i/\partial c\partial d = -e^d(t_i-b)\phi_{t_i}\psi_{t_i}.$$

References

Birnbaum, A. (1968). Some latent trait models and their use in inferring an examinee's ability. In F.M. Lord & M.R. Novick (Eds.), *Statistical theories of mental test scores.* Reading, MA: Addison–Wesley.

Bock, R.D., & Aitken, M. (1981). Marginal maximum likelihood estimation of item parameters: An application of an EM algorithm. *Psychometrika*, *46*, 443–459.

Box, G.E.P., & Tiao, G. C. (1973). *Bayesian inference in statistical analysis.* Reading, MA:Addison–Wesley.

Dempster, A.P., Laird, N.M., & Rubin, D.B. (1977). Maximum likelihood from incomplete data via the EM algorithm (with discussion). *Journal of the Royal Statistical Society*, Series B, 39, 1–38.

Ferguson, T.S. (1973). A Bayesian analysis of some nonparametric problems. *Annals of*

Statistics, 1, 209–230.

Lindley, D.V., & Smith, A.F.M. (1972). Bayes estimates for the linear model (with discussion). *Journal of the Royal Statistical Society, Series B*, 34, 1–41.

Lord, F.M. (1980). *Applications of item response theory to practical testing problems.* Hillsdale, NJ:Erlbaum.

Lord, F.M. (1986). Maximum likelihood and Bayesian parameter estimation in item response theory. *Journal of Educational Measurement*, 23, 157–162.

Marquardt, D.W. (1963). An algorithm for least–squares estimation of non–linear parameters. *Journal of the Society for Industrial and Applied Mathematics*, 11, 431–441.

Mislevy, R.J. (1986). Bayes modal estimation in item response models. *Psychometrika*, 51, 177–195.

Mislevy, R.J., & Bock, R.D. (1984). *BILOG: Item analysis and test scoring with binary logistic models.* Mooresville, IN: Scientific Software.

Naylor, J.C., & Smith, A.F.M. (1982). Applications for efficient computation of posterior distributions. *Applied Statistics*, 31, 214–225.

Ramsey, F.L. (1972). A Bayesian approach to bioassay, *Biometrics*, 28, 841–58.

Rigdon, S.E., & Tsutakawa, R.K. (1983). Estimation in latent trait models. *Psychometrika* 48, 567–574.

Swaminatham, H., & Gifford, J.A. (1986). Bayesian estimation in the three–parameter logistic model. *Psychometrika*, 51, 589–601.

Tsutakawa, R.K. (1984). Estimation of two–parameter logistic item response curves. *Journal of Educational Statistics*, 9, 263–276.

Tsutakawa, R.K., & Lin, H.Y. (1986). Bayesian estimation of item response curves. *Psychometrika*, 51, 251–267.

Tsutakawa, R.K., & Soltys, M. (1988). Approximation for Bayesian ability estimation. *Journal of Educational Statistics*, in press.

Wilks, S.S. (1962). *Mathematical Statistics.* New York:Wiley.

Wingersky, M.S., Barton, M.A., & Lord, F.M. (1982). *LOGIST user's guide.* Princeton, NJ:Educational Testing Service.

TABLE 1

Summary of Dirichlet (D) and Simulated Constrained Dirichlet (CD) Priors

Distribution	N	$\bar{p}_1$	$\bar{p}_2$	$\bar{p}_3$	s_1	s_2	s_3
ACT81		.217	.441	.835	.100	.180	.135
D	3	.217	.441	.835	.206	.248	.186
CD	3	.243	.389	.866	.204	.227	.163
D	9.8	.217	.441	.835	.125	.151	.113
CD	9.8	.227	.415	.847	.122	.142	.110
D	24	.217	.441	.835	.082	.099	.074
CD	24	.226	.432	.841	.082	.092	.071

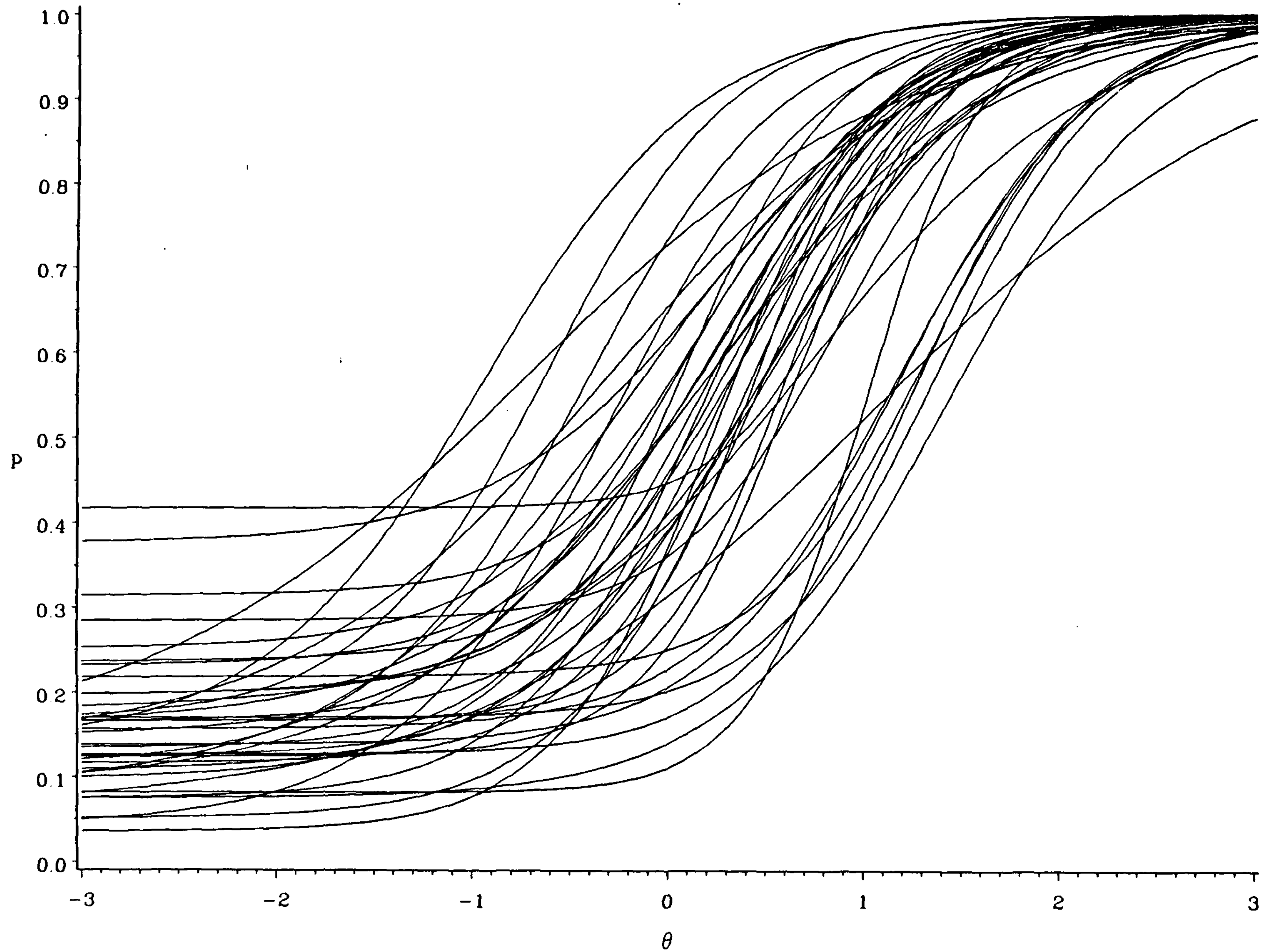

FIGURE 1
3PL curves for 1981 ACT test estimated by LOGIST.

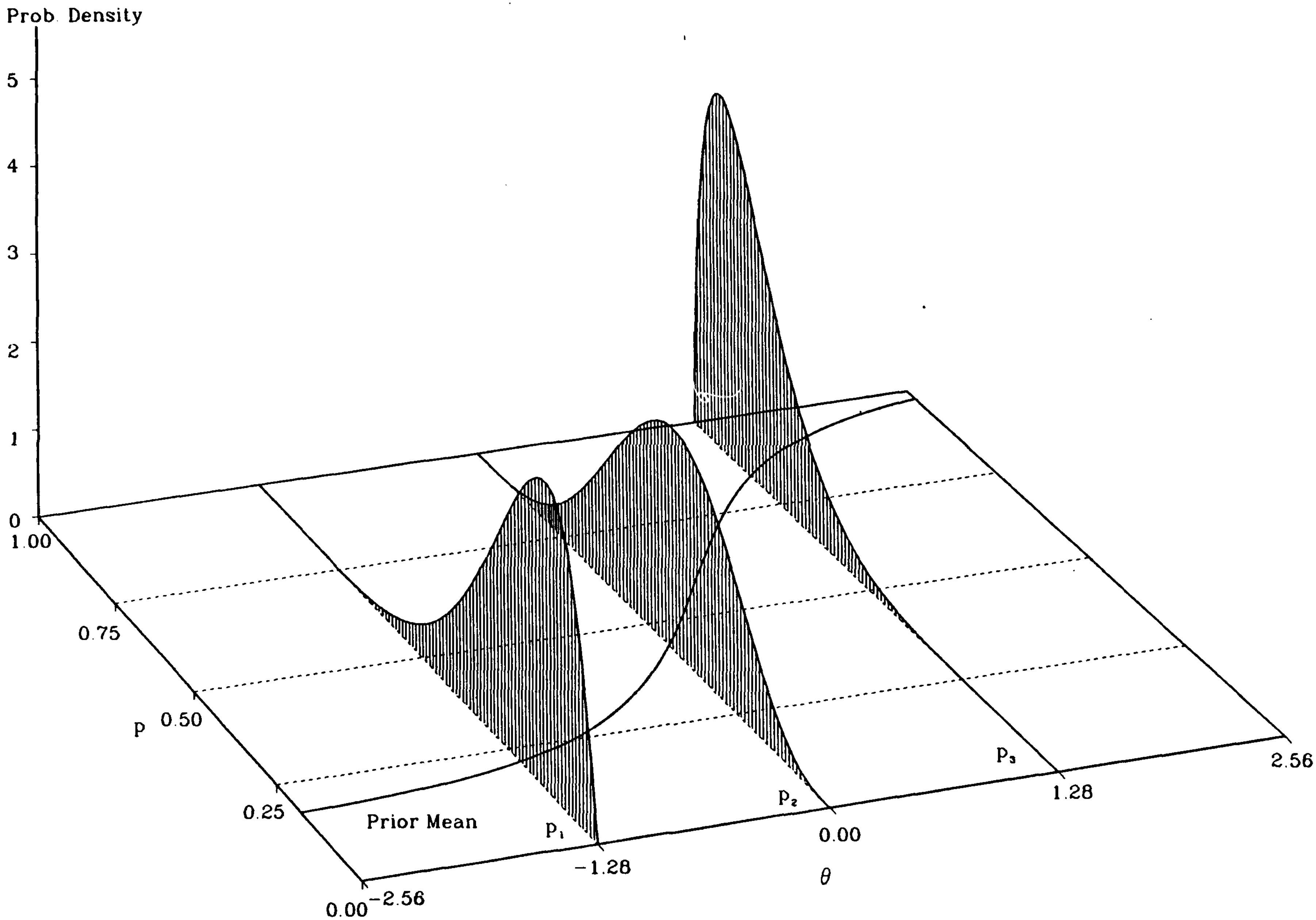

FIGURE 2
Prior mean and marginal pdf's of p_1, p_2, and p_3.

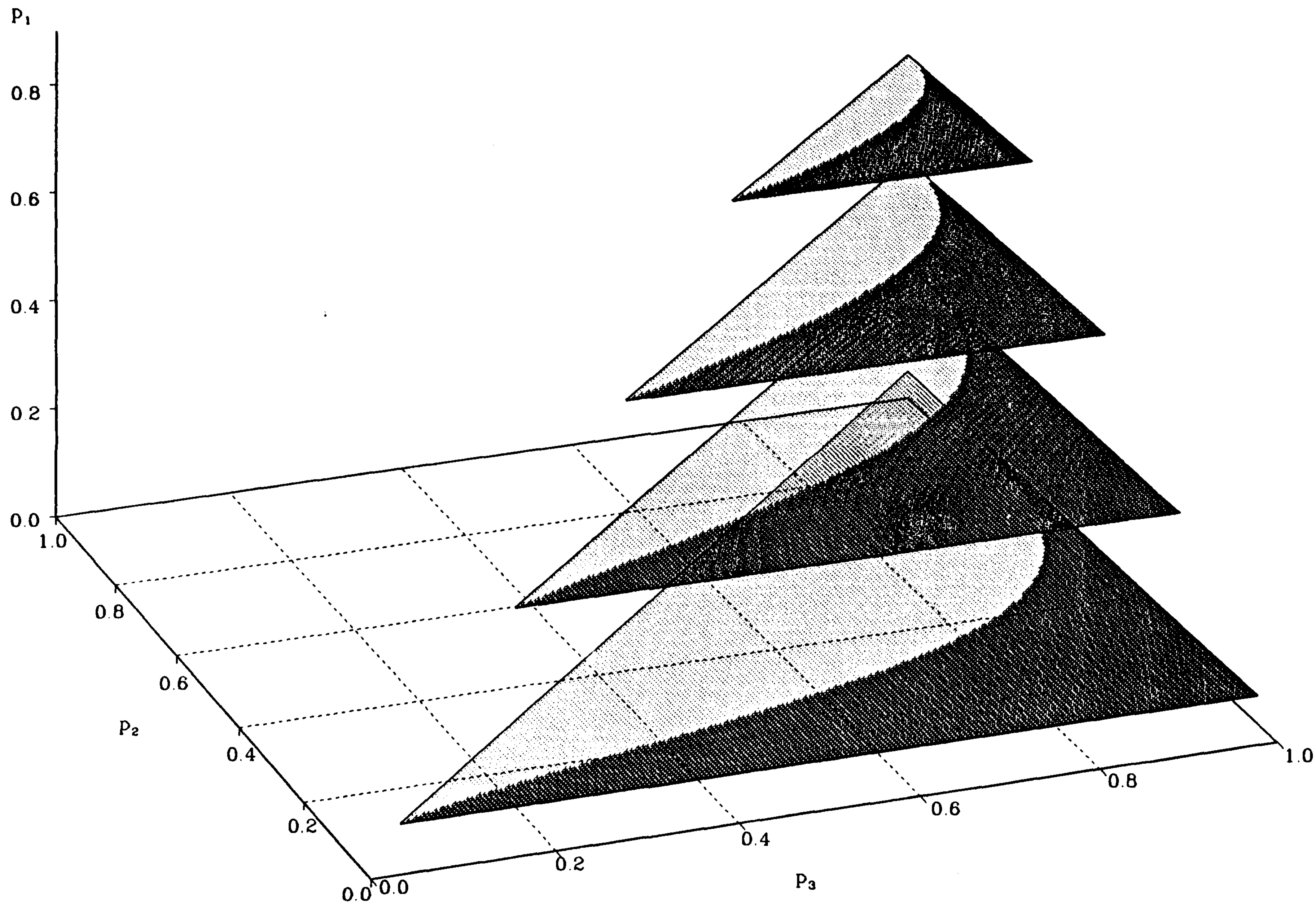

FIGURE 3
Sample spaces under Dirichlet and constrained Dirichlet (dark shaded area).

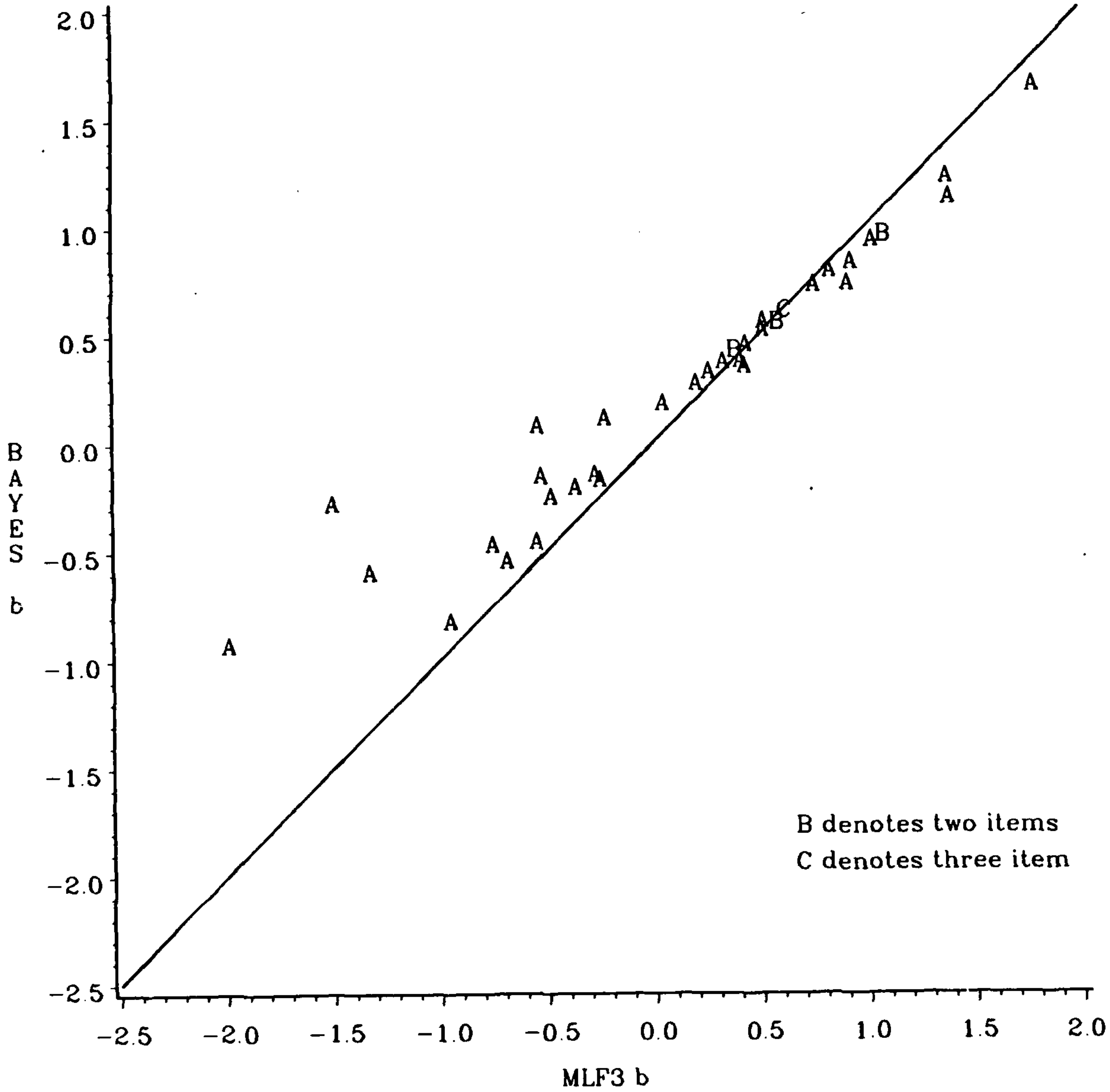

FIGURE 4
Bayes vs. MLF3 estimates of b.

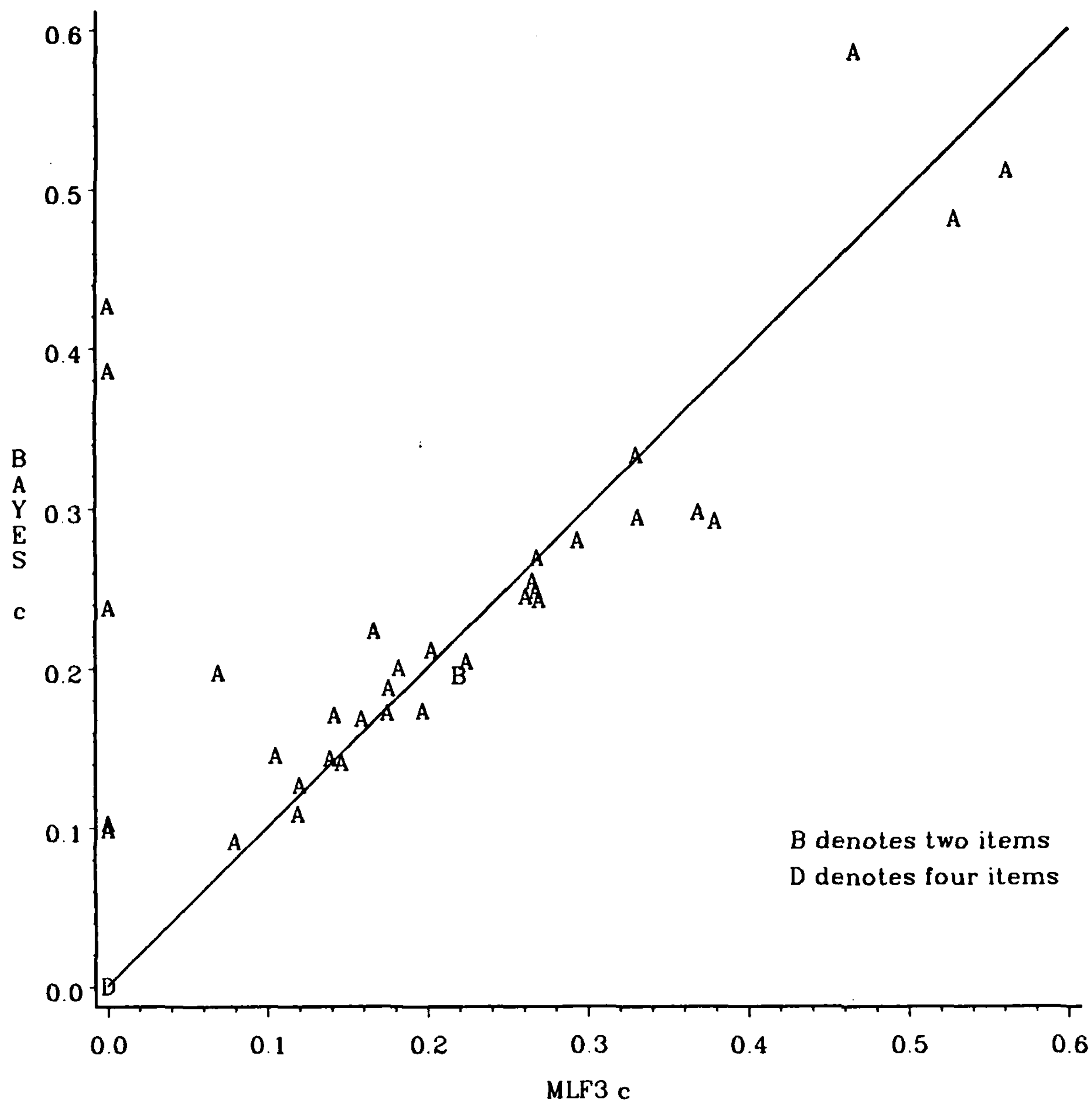

FIGURE 5
Bayes vs. MLF3 estimates of c.

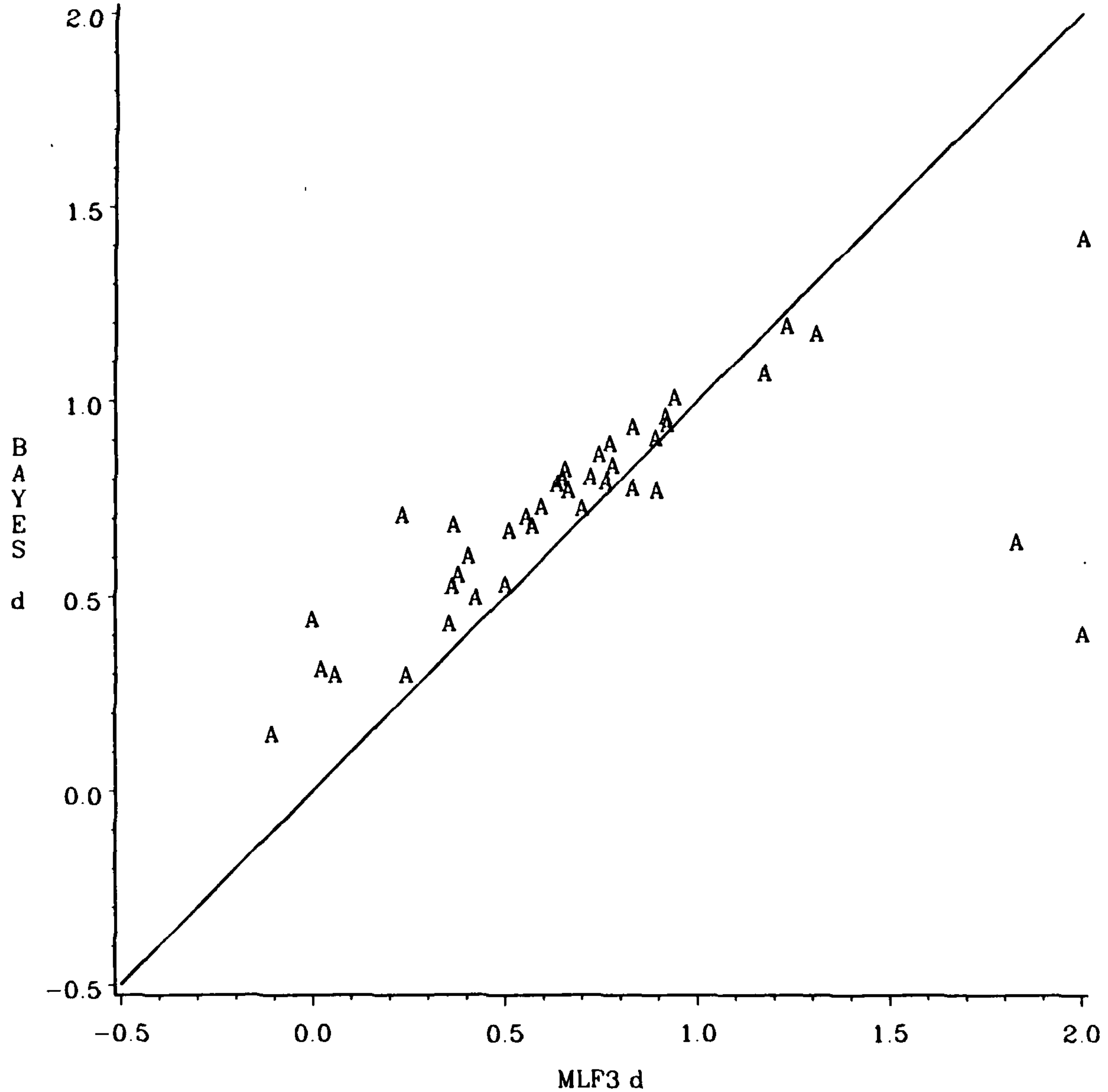

FIGURE 6
Bayes vs. MLF3 estimates of d.

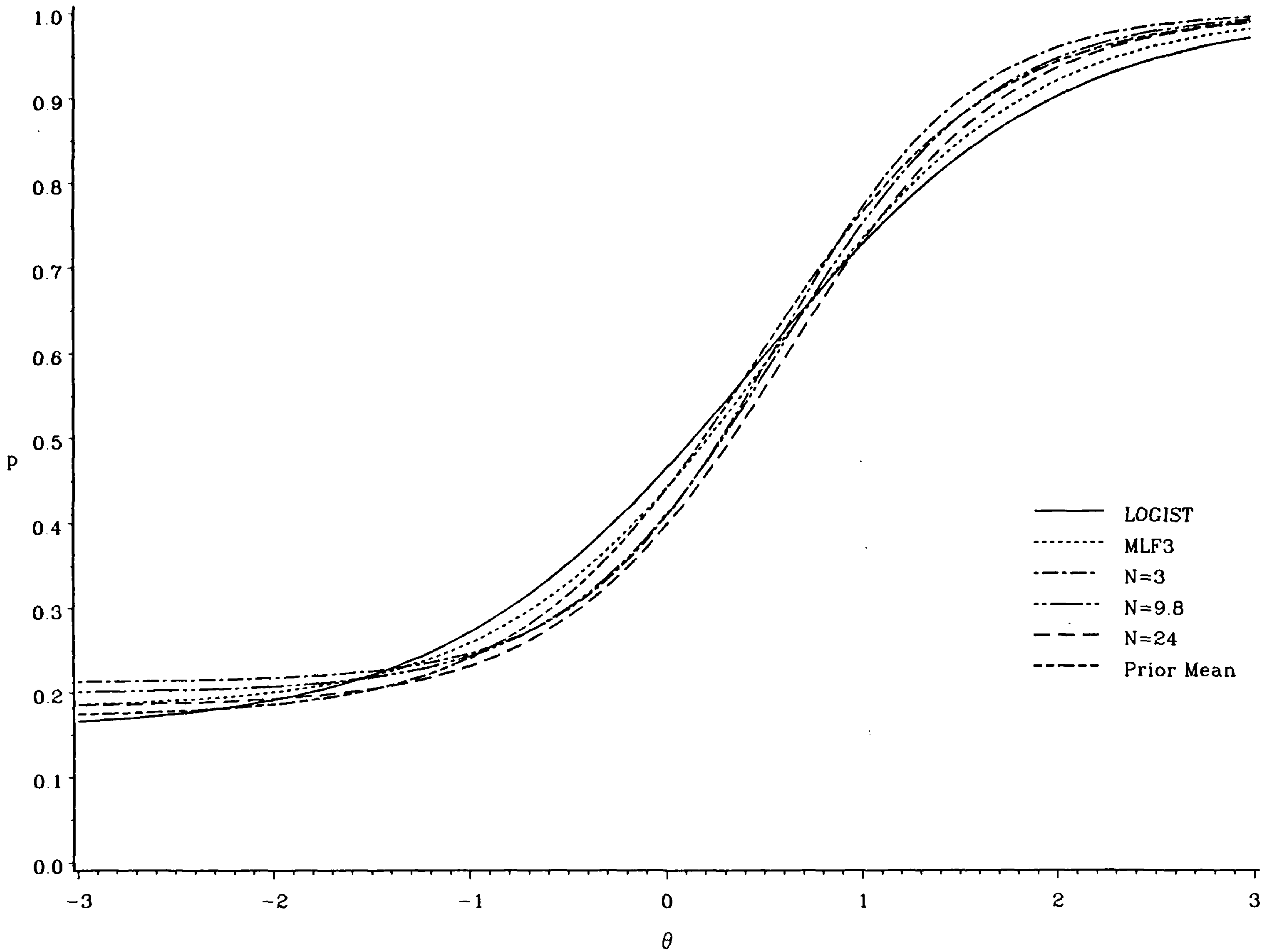

FIGURE 7
Estimated 3PL curves for item 27

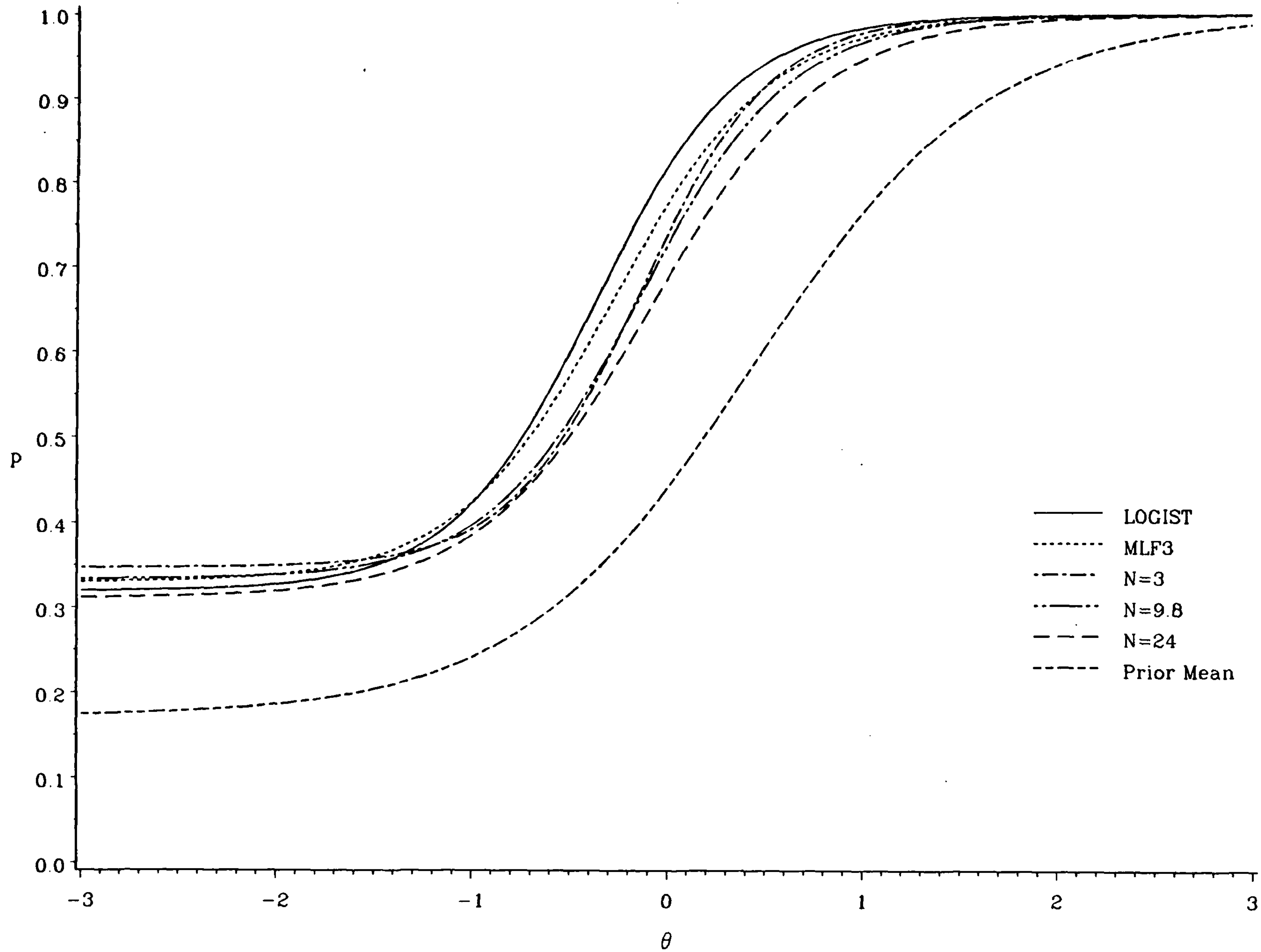

FIGURE 8
Estimated 3PL curves for item 7

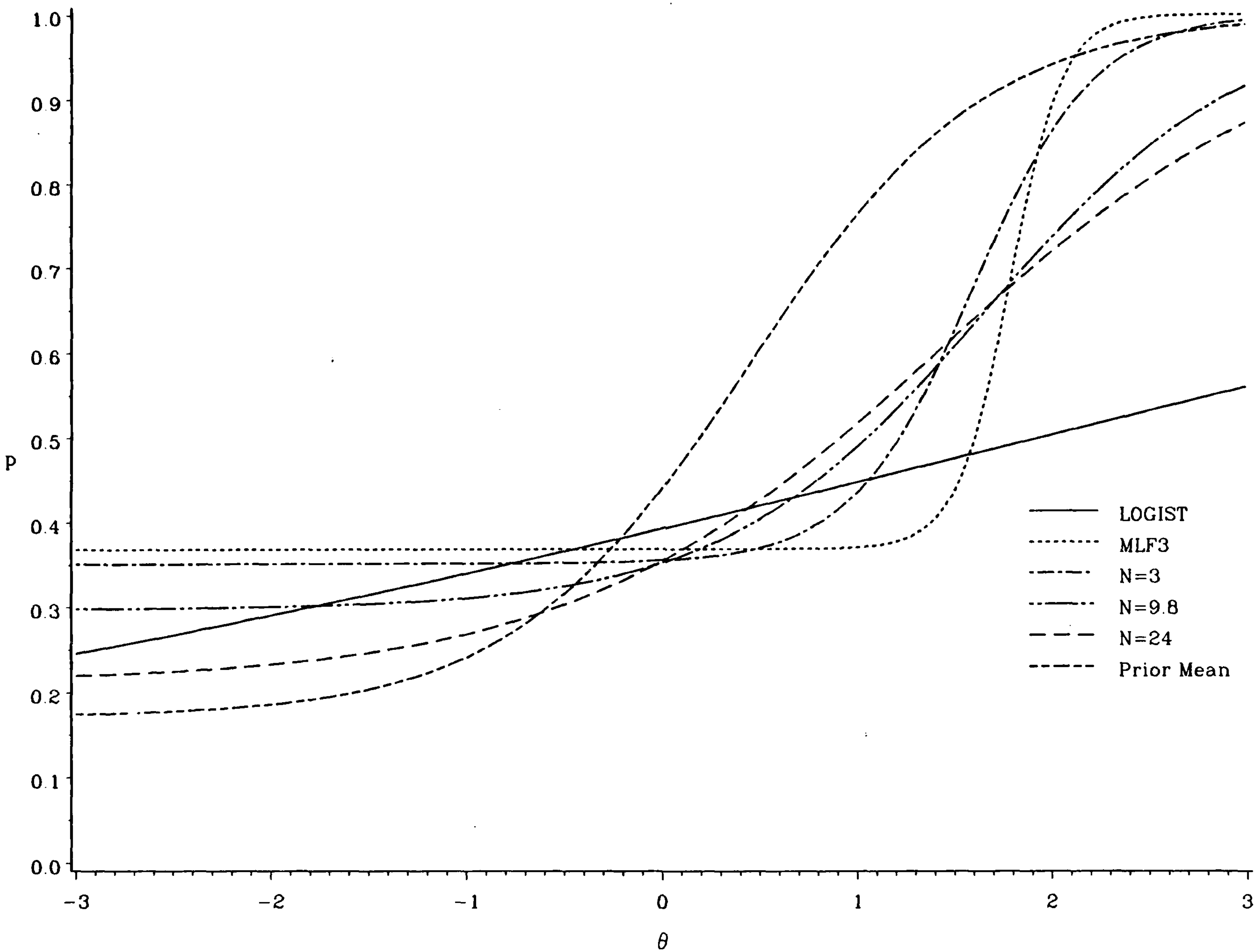

FIGURE 9
Estimated 3PL curves for item 13

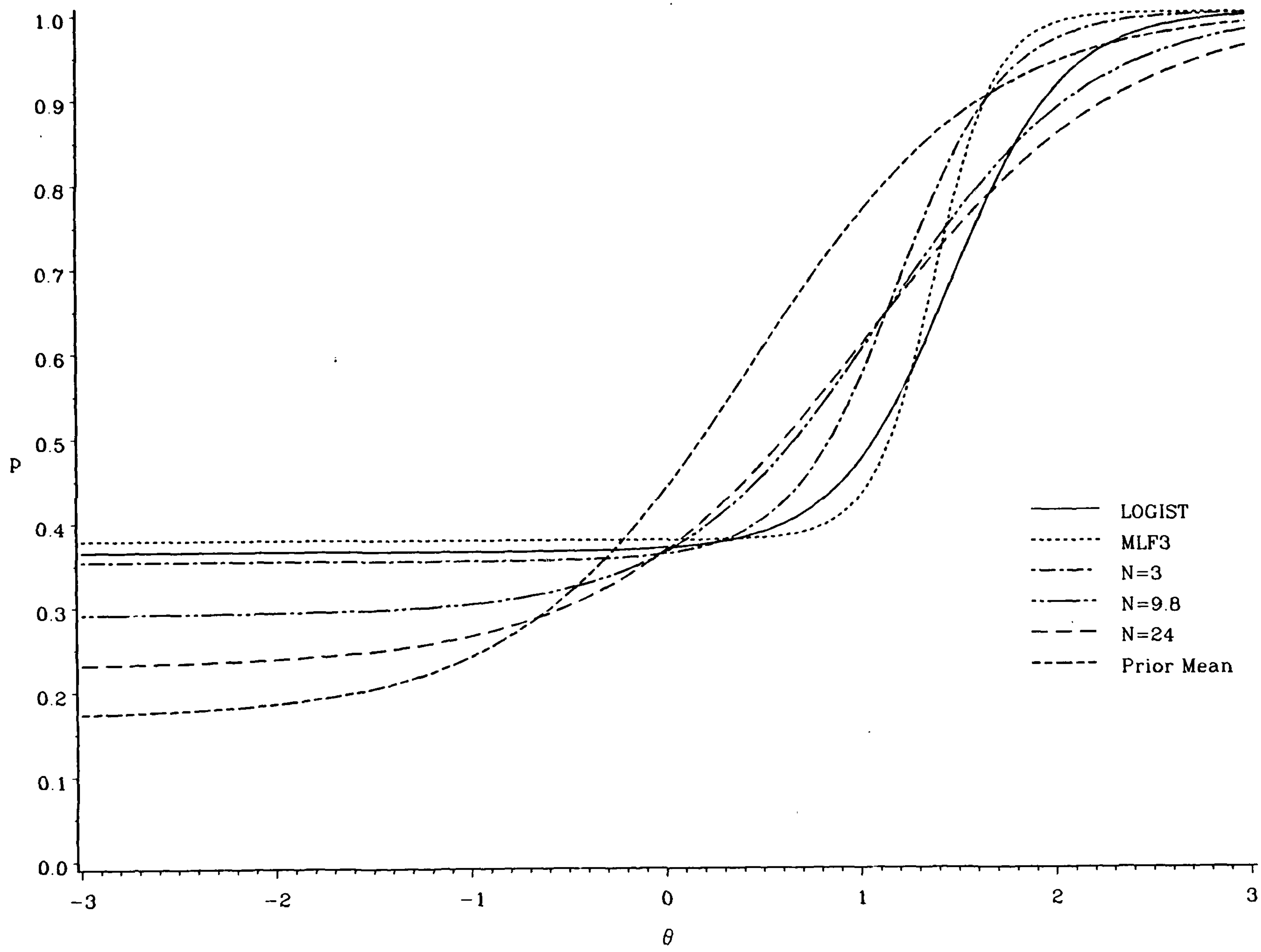

FIGURE 10
Estimated 3PL curves for item 22

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